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CITATION:

Kimura, Masayuki ...[et al]. A study on bifurcations and structure of phase space concerning intrinsic localized modes in a nonlinear magneto-mechanical lattice. AIP Conference Proceedings: NONLINEAR ACOUSTICS State-of-the-Art and Perspectives 2012, 1474: 55-59

ISSUE DATE:

2012-09-04

URL:

http://hdl.handle.net/2433/187945

A study on bifurcations and structure of phase space concerning intrinsic localized modes in a nonlinear magneto-mechanical lattice

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Abstract. A magneto-mechanical lattice proposed in this paper is one of the nonlinear lattices in which intrinsic localized mode(ILM) exists. A bifurcation diagram concerning ILMs is investigated with respect to the magnitude of nonlinear coupling force. In addition, the possibility of the existence of moving ILM is discussed based on the phase structure around an unstable ILM which is numerically examined by computing unstable manifolds of the unstable ILM.

Keywords: intrinsic localized mode, discrete breather, cantilever array, moving ILM
PACS: 05.45.-a, 63.20.Pw, 63.20.Ry

INTRODUCTION

Intrinsic localized mode(ILM) is an energy localized phenomenon appeared in nonlinear lattices [1]. In this decade, ILM was identified not only in natural structures such as antiferromagnet [2] but also in artificial structures, for instance, micro-mechanical cantilever array [3], electronic circuits [4, 5], and magneto-mechanical lattice [6]. The experiments in the micro-mechanical cantilever arrays allow us to expect the realization of applications using the energy localization in micro/nano-engineering because ILM can move without decaying its energy concentration and can be manipulated by an extraneous stimulus [7]. However, it is needed to realize such application that the mechanism of how ILM moves should be clarified and the control scheme has to be established.

We have proposed the magneto-mechanical lattice to investigate the dynamics of ILM [8]. Although standing ILMs were successfully observed and manipulated in the lattice, any moving ones could not be generated. In this paper, an improved magneto-mechanical lattice which have a nonlinearity in coupling force is first introduced. Then, bifurcations and the phase space around ILMs are investigated numerically. The possibility of the existence of moving ILM is finally discussed based on the phase structure.

MAGNETO-MECHANICAL LATTICE

A cantilever which behaves as a linear oscillator for small deflection is used as an oscillator of the magneto-mechanical lattice. A small magnet is attached at the tip of

NONLINEAR ACOUSTICS State-of-the-Art and Perspectives
AIP Conf. Proc. 1474, 55-58 (2012); doi: 10.1063/1.4749297

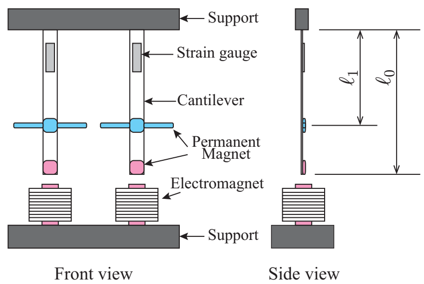


FIGURE 1. Configuration of magneto-mechanical lattice

the cantilever and an electromagnet is placed beneath the tip to cause a nonlinearity in the restoring force of the cantilever. For the coupling force between adjacent cantilevers, another magnet is sticked at the middle of each cantilever. The configuration of these magnets are shown in Fig. 1.

By using the magnetic charge approximation for the magnetic interactions [6], the motion of equation for the magneto-mechanical lattice is obtained as follows:

$$\ddot{u}_n = -\omega_0^2 u_n - \chi_{\mathrm{O}} \frac{u_n}{(u_n^2+d_1^2)^{3/2}} - \chi_{\mathrm{I}} \frac{u_n - u_{n+1}}{\left\{(u_n-u_{n+1})^2 + d_2^2/\kappa^2\right\}^{3/2}} - \chi_{\mathrm{I}} \frac{u_n - u_{n-1}}{\left\{(u_n-u_{n-1})^2 + d_2^2/\kappa^2\right\}^{3/2}}, \tag{1}$$

where u_n is the displacement of the tip of nth cantilever. Gaps between on-site and inter-site magnets are denoted by $d_1 (= 3.0\,\mathrm{mm})$ and $d_2 (= 2.0\,\mathrm{mm})$, respectively. χ_{O} and χ_{I} are coefficients of magnetic force. Since the current flowing in electromagnet determines the magnitude of magnetic flux on the surface, χ_{O} can be tuned dynamically. The resonant frequency of cantilever is represented by $\omega_0 = 2\pi f_0 (= 2\pi \times 35.0\,\mathrm{rad/s})$. The ratio of the position of inter-site magnet to the length of cantilever is denoted by $\kappa = \ell_1/\ell_0 (= 50\,\mathrm{mm}/70\,\mathrm{mm} = 5/7)$.

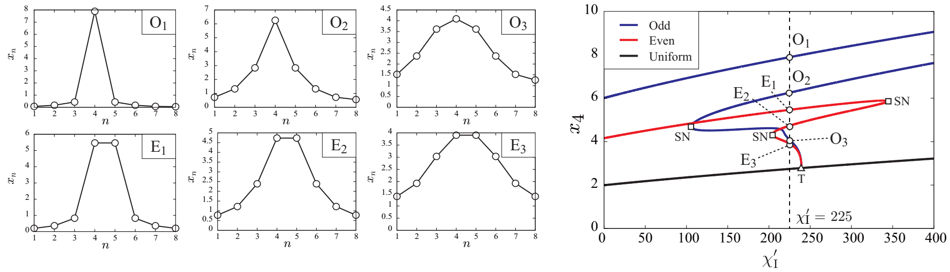
For simplicity and generality, Eq.(1) is nondimensionalized by substituting $t \to tT_0$ and $u_n \to x_n U_0$.

$$\ddot{x}_n = -(2\pi)^2 x_n - \chi'_{\mathrm{O}} \frac{x_n}{\left(x_n^2 + {d'_1}^2\right)^{3/2}} - \chi'_{\mathrm{I}} \frac{x_n - x_{n+1}}{\left\{(x_n - x_{n+1})^2 + {d'_2}^2\right\}^{3/2}} - \chi'_{\mathrm{I}} \frac{x_n - x_{n-1}}{\left\{(x_n - x_{n-1})^2 + {d'_2}^2\right\}^{3/2}}, \tag{2}$$

where $x_n = u_n/U_0$, $\chi'_{\mathrm{O}} = \chi_{\mathrm{O}}/U_0^3 f_0^2$, $\chi'_{\mathrm{I}} = \chi_{\mathrm{I}}/U_0^3 f_0^2$, $d'_1 = d_1/U_0$, $d'_2 = d_2/U_0\kappa$. In this paper, the scale parameters are set to $T_0 = 1/f_0 \simeq 28.57\,\mathrm{ms}$ and $U_0 = 1\,\mathrm{mm}$.

BIFURCATIONS OF INTRINSIC LOCALIZED MODES

In this section, bifurcations of standing ILMs whose location of the center of energy concentration is fixed are investigated. Since Eq. (2) has no damping term and no forcing term, the total energy(E_{tot}) is conserved and is a bifurcation parameter. The magnitude of magnetic coupling between adjacent cantilevers χ'_{I} is also a bifurcation parameter. In this paper, we investigate bifurcations with respect to χ'_{I} instead of E_{tot}. In Fig. 2(a), amplitude distribution is shown for coexisting ILMs at $\chi'_{\mathrm{I}} = 255$, $E_{\mathrm{tot}} = -100$. These



(a) Coexisting ILMs

(b) Bifurcation diagram

FIGURE 2. Amplitude distribution of ILM and bifurcation diagram

ILMs can be classified into two kinds by spatial symmetry of the amplitude distribution. The odd-symmetric ILMs are labeled O_1, O_2, and O_3 while the even-symmetric ones are shown by E_1, E_2, and E_3. The bifurcation diagram for the ILMs is shown in Fig. 2(b). The open squares correspond to saddle-node bifurcation points. ILMs having the same symmetry appear or disappear at the bifurcation points. On the other hand, an odd-symmetric ILM and an even-symmetric ILM coalesce with the uniform vibration at the tangent bifurcation point [9] indicated by the open triangle. The tangent bifurcation point is only one bifurcation point related to the two different kinds of ILM in the bifurcation diagram. In the next section, the phase structure around coexisting ILMs near the tangent bifurcation point is discussed.

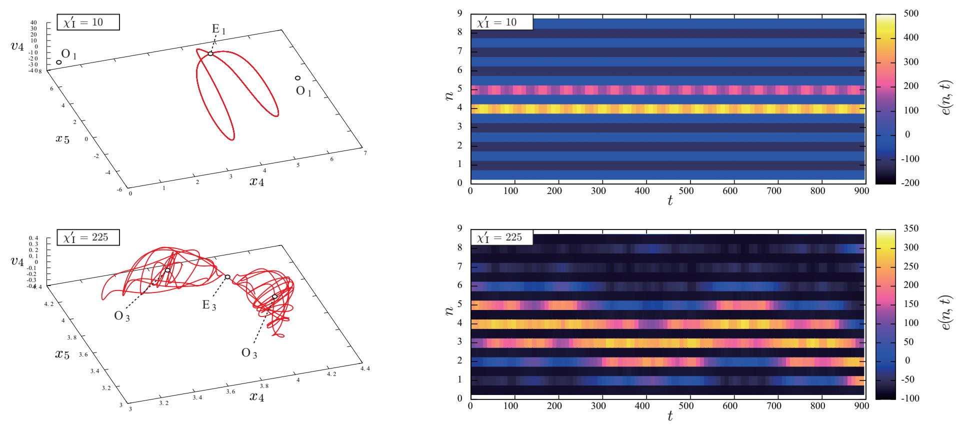
PHASE STRUCTURE AND MOVING ILM

To investigate the phase structure around coexisting ILMs, an one-dimensional unstable manifold of unstable ILM is computed. In the upper panel of Fig. 3(a), the unstable manifold of the unstable even-symmetric ILM(E_1) is shown for the case of weak coupling regime $\chi_I' = 10$. The invariant manifold shows a homoclinic-like structure, but does not reach to the neighboring odd-symmetric ILMs(O_1). On the other hand, the invariant manifold of E_3 surrounds the neighboring odd-symmetric ILMs(E_3) for the case of near the tangent bifurcation point $\chi_I' = 255$.

The tendency of the behavior of the solution started from near the unstable ILM can be estimated because the structure of the invariant manifold in the phase space reflects the flow of Eq.2 [10]. Therefore, if the structure shows connections between two different ILMs, the existence of moving ILM can be expected. In fact, as shown in the lower panel of Fig. 3(b), the time development of the energy distribution of a solution whose initial condition is crated by slightly perturbing E_3 shows a wandering locus of the energy concentration, namely, a moving ILM.

CONCLUSION

The magneto-mechanical lattice which has nonlinearities in both on-site and inter-site potentials was introduced and modeled as the coupled ordinary differential equations.



(a) Unstable manifold

(b) Time development of energy distribution

FIGURE 3. Unstable manifolds and time development of energy distribution of the perturbed ILMs

For the nondimensionalized equation, bifurcations of coexisting ILMs were investigated with respect to the coupling force. The saddle-node bifurcations and the tangent bifurcation was observed in the bifurcation diagram. The phase structure near ILMs was examined by computing the unstable manifold of the unstable ILM for two cases, one was in the weak coupling regime and the other was near the tangent bifurcation. It was revealed that the unstable manifold surrounds the neighboring ILMs for the latter case. The fact that the moving ILM was easily created implies that the existence of moving ILM can be expected near a bifurcation point related to different symmetric ILMs such as the tangent bifurcation point.

ACKNOWLEDGMENTS

This work was supported by the Ministry of Education, Culture, Sports, Science and Technology in Japan, Grant-in-Aid for Young Scientist (B) No. 22760280.

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